

# **A Novel method for Reliability-Based Design Optimization under the fatigue life constraints by using the outcrossing approach.**

Younes AOUES, Leila KHALIJ, Emmanuel PAGNACCO, Didier LEMOSSE.

*INSA Rouen-LOFIMS, Laboratory of optimisation and reliability in structure mechanics, 685 avenue de l'université, 76801 Saint Etienne de Rouvray, France.*

## **Abstract:**

Mechanical structures subjected to complex random time-varying stresses may suffer of severe damages. The classical deterministic approaches for high-cycle fatigue life prediction are not sufficient for the design of these structures. Indeed, the uncertainties in loadings involve a probabilistic prediction of the fatigue life. Furthermore, for design economical and reliable structures under the probabilistic fatigue life constraints, a strategy involving the Reliability Based Design Optimization (RBDO) has to be applied.

The RBDO problem involves the evaluation of probabilistic constraints performed by the reliability analysis, which can be done by stochastic simulations or by moment methods. Monte Carlo simulations being computationally expensive, the moment methods are widely used in RBDO procedures due to their simplicity and efficiency [1], especially the first order reliability method (FORM). The most common RBDO formulations are based on a nested optimization problem called the two-level RBDO approach, with an outer loop for design optimization and an inner loop for reliability analysis. Therefore, an expensive computational effort is required to solve the RBDO problem. In order to overcome computational difficulties, many formulations have been recently developed, as the single loop and the decoupled approaches [2]. However, RBDO approaches are still a difficult task since it requires the coupling of optimization algorithms, reliability analysis and mechanical modeling. This situation becomes impracticable for complex structures when finite element analysis considering nonlinear material behavior, dynamic analysis and fatigue life prediction analysis are involved.

Several works are focused on developing efficient and robust approaches for RBDO problem [2]. However, few studies are interested by Time-Variant Reliability-Based Design Optimization (TV-RBDO). The TV-RBDO approach aims in finding the optimal design by satisfying appropriate safety level during the whole structure lifetime by minimizing an objective function defined by the initial and expected failure costs under time-variant reliability constraints [3]. The time variant reliability analysis consists in computing the probability of failure during the whole structure lifetime, when the time dependency lies in the loading and the degradation phenomena. Several approaches are addressed to assess the time-variant reliability, which can be done either by simulation techniques or by approximate approaches. Sampling methods such as the Monte Carlo simulation are computationally very expensive. The approximate approaches are generally based on the outcrossing approach [4], which relates to the mean number of outcrossing of the random process through the limit state surface [5].

While the RBDO is a very active research field in these last decades and several authors have been proposed reliability methods using a random approach for the fatigue life prediction [6-10], only few works consider the fatigue life function in RBDO problems [11-13]. The originality of this work is to consider the time-variant reliability based design optimization under the fatigue constraints by using the spectral approach for the fatigue life prediction and the outcrossing method for the time variant reliability analysis.

This paper focuses on a probabilistic life prediction methodology based on random process representation computationally efficient for assessing the probability of failure on the basis of the Rice Formula [14] for the outcrossing rate. The numerical example demonstrates the effectiveness of the proposed approach to consider the probabilistic fatigue life constraints in the TV-RBDO formulation by using the spectral approach for the fatigue life prediction

## **References**

- [1] Frangopol, D.M & Moses, F. 1994. *Reliability-based structural optimization*. Adeli H, editor. Advances in design optimization. London:Chapman Hall.

- [2] Y. Aoues, A. Chateaneuf. Benchmark study of numerical methods for reliability-based design optimization, *Structural & Multidisciplinary optimization* 2010;41(2):277–294.
- [3] Kuschel, N & Rackwitz, R. 2000. Optimal design under time-variant reliability constraints. *Structural Safety* 22(2): 113-27.
- [4] Ditlevsen, O & Madsen, H.O. 1996. *Structural reliability method*. New York : John Wiley and Sons.
- [5] Hagen, O & Tvedt, L. 1991. Vector Process Out-Crossing as Parallel System Sensitivity Measure. *Journal of Engineering Mechanics* 117(1): 2201-20.
- [6] Y. Liu, S. Mahadevan. Efficient methods for time-dependent fatigue reliability analysis. *AIAA Journal* 2009;47(3):494–504.
- [7] L. Yu. Fatigue reliability of ship structures. PhD thesis, university of Glasgow 2010.
- [8] H. Karadeniz. Uncertainty modeling in the fatigue reliability calculation of offshore structures. *Reliability Engineering & System Safety* 2001 ; 74(3):323–335
- [9] J. Tang, J. Zhao. A practical approach for predicting fatigue reliability under random cyclic loading. *Reliability Engineering & System Safety* 1995 ; 50(1):7–15
- [10] S. Lambert, E. Pagnacco, L. Khaliq. A probabilistic model for the fatigue reliability of structures under random loadings with phase shift effects. *Int. J. of Fatigue* 2010;32:463–474.
- [11] R. d'Ippolito, M. Hack, S. Donders, L. Hermans, N. Tzannetakis, D. Vandepitte. Improving the fatigue life of a vehicle knuckle with a reliability-based design optimization approach. *J. of Statistical Planning and Inference* 2009;139(5):1619 – 1632.
- [12] X. Yu, K.K. Choi, K.H. Chang. A mixed design approach for probabilistic structural durability, *Structural optimization* 1997;14(3-4):81–90.
- [13] K. Choi, J. Tang, E. Hardee, B. Youn. Application of Reliability-Based Design Optimization to Durability of Military Vehicles. *SAE Technical Paper* 2005, 2005-01-0530.
- [14] Rice S. Mathematical analysis of random noise part I and II. *Bell System Technical Journal*, 32 :282–332, 1944.